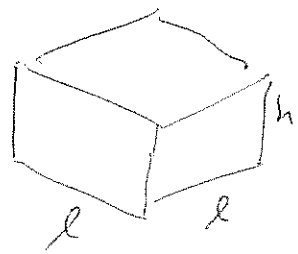
9:06

Quiz 6A - MTH 1410

Name: Key

1) (4 points) New postal regulations stipulate that for any box with a square base, the sum of the length and the height can be at most 7 feet. Use calculus to find the maximum volume for such a box. Round your answer to the nearest two decimal places. Make sure to confirm your location is a max.



$\ell + h = 7$

$h = 7 - \ell$

domain of $\ell = [0, 7]$

$V = \ell^2 h$

$V(\ell) = \ell^2(7 - \ell)$

$V(\ell) = 7\ell^2 - \ell^3$

$V' = 14\ell - 3\ell^2$

$V' = \ell(14 - 3\ell)$

$V' = 0$ when $\ell = 0$

or $14 - 3\ell = 0$

$\ell = \frac{14}{3}$

$V(0) = 0$

$V(7) = 0$

$V\left(\frac{14}{3}\right) = \boxed{50.81}$

is a max

2) (4 points) Find the following limit. If the limit does not exist, explain why.

$$\lim_{x\to\infty} 3x\tan\left(\frac{1}{x}\right)$$

$$= \lim_{x\to\infty} \frac{3\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} \quad \to \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{3\sec^2\left(\frac{1}{x}\right)\cdot\frac{-1}{x^2}}{\frac{-1}{x^2}}$$

$$= 3\sec^2(0)$$

$$= \boxed{3}$$

3) (2 points) Approximate a solution to the equation $x^2 = 5x - 2$. Given $x_1 = 1$, use Newton's Method to find x_3. Round to nearest 2 decimal places.

$x^2 - 5x + 2 = 0$

$f(x) = x^2 - 5x + 2$

$f'(x) = 2x - 5$

$x_1 = 1$

$$x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-2}{-3} = \frac{1}{3}$$

~~$x_3 = \frac{5}{3} - \frac{f(5/3)}{f'(5/3)} = 1.166 - 3.555 = -1.666$~~

$$x_3 = \frac{1}{3} - \frac{f(1/3)}{f'(1/3)} = \frac{1}{3} - \frac{0.4444}{-4.333} = 0.435897$$

≈ 0.44